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ABSTRACT

15790 author

The concept of a free-piston shock tube driver is discussed in detail and an analysis is made of the heat losses encountered in the compression process. Design optimization is discussed and methods of controlling driver performance are outlined in detail. Author

I. INTRODUCTION

Within the past few years much effort was expended toward the creation of high stagnation enthalpy and high temperature shock-tube flows. Conventional drivers, such as the cold pressurized and combustion drivers, have fallen short of the required temperature and pressure capability, and several alternate driving methods have been proposed. To date, however, only the electric-arc driver has been put into widespread use (Ref. 1).

The electric-arc driver possesses several undesirable characteristics. The arcing process creates a narrow core of highly ionized hot gas within an outer shell of cooler gas, resulting in a large radial nonuniformity. Furthermore, the insulation required for the driver walls ablates when the arc is struck and contaminates the driver gas. Since the effect of these nonuniformities is not fully understood and a complete understanding does not appear imminent, an analysis has been made of an alternate method, free-piston compression, of driving a high enthalpy shock tube.

The free-piston compression technique was first applied to driving a shock tube by Stalker (Ref. 2) and the technique was further developed by Bryson (Ref. 3). It consists of the isentropic compression of the driver gas by a free piston moving in a tube, depicted schematically in Fig. 1. The apparatus is divided into three sections: the reservoir, the piston tube, and the shock-tube driver-

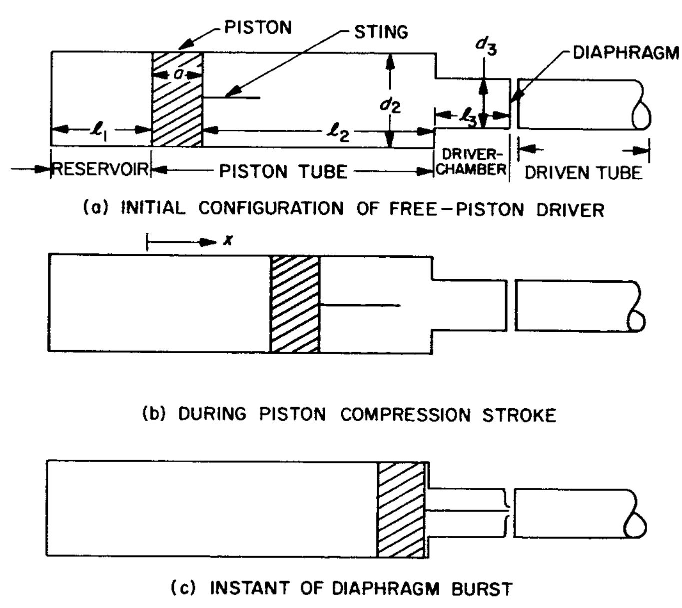

Fig. 1. Schematic of free-piston driver operation

chamber. The piston-tube driver-chamber combination is filled with the shock-tube driver gas at low pressure. The reservoir is filled with the piston-driving gas at high pressure while the piston is held at the rear of the piston tube by a mechanical triggering device. When the piston is released it travels forward, compressing the driver gas ahead of it. The initial pressure ratio across the piston can be adjusted to a value which will allow the piston to just reach the rear of the driver-chamber before the force of the compressed driver gas reverses its direction. At that instant, a sting, mounted on the front face of the piston, pierces the diaphragm and initiates the shock-tube flow.

The most serious questions raised regarding the feasibility of the technique were those concerning heat loss during the compression stroke and the effect of the short driver, if the overall size of the device is to be kept to reasonable proportions. The motion of the piston is analyzed in section II of this Report, and the analysis is utilized in section III to treat the heat loss problem. Here, a set of design rules is evolved and an estimate is made of the heat loss to be expected from one particular design. The remainder of the Report deals with the operation of the device, the performance to be expected, and several means of increasing performance of a fixed facility.

II. PISTON MOTION

A. Equation of Motion

The force on the piston at some time during its stroke is

$$F = \frac{\pi d_1^2}{4} p_1 - \frac{\pi d_2^2}{4} p_2 - f \tag{1}$$

where f is the frictional force; p_1 and p_2 are the pressures of the piston-driving gas and the driver gas, respectively.

In general, a gas compression or expansion process can be characterized by the polytropic relation

$$p \sim \rho_g^n \tag{2}$$

If there were no losses during the process, $n = \gamma$; as energy is lost, n will decrease.

Assuming the piston velocity to be considerably subsonic and using Eq. (2), pressures p_1 and p_2 can be expressed in terms of the system geometry (Fig. 1) as

$$p_1 = p_{0_1}\left(\frac{l_1}{l_1 + x}\right)^{n_1} \tag{3}$$

and

$$p_2 = p_{0_2}\left[\frac{l_3 d_3^2 + l_2 d_2^2}{l_3 d_3^2 + (l_2 - x) d_2^2}\right]^{n_2} \tag{4}$$

where the subscript 0 refers to initial conditions, i.e., $x = 0$. Finally, the mass of a solid piston is

$$m = \frac{\pi \rho d_2^2 a}{4g}$$

Now, from Newton's law, it follows that

$$\frac{d^2x}{dt^2} = \frac{g}{\rho d_2^2 a}\left\{d_1^2 p_{0_1}\left(\frac{l_1}{l_1 + x}\right)^{n_1} - d_2^2 p_{0_2}\left[\frac{l_3 d_3^2 + l_2 d_2^2}{l_3 d_3^2 + (l_2 - x) d_2^2}\right]^{n_2} - \frac{4f}{\pi}\right\} \tag{5}$$

Introducing a nondimensional friction parameter

$$\phi = \frac{4f}{\pi d_2^2 \rho a}$$

and a nondimensional pressure parameter

$$\bar{p}_i = \frac{p_i}{\rho a}$$

Eq. (5) becomes

$$\frac{1}{g}\frac{d^2x}{dt^2} = \bar{p}_{0_1}\left(\frac{l_1}{l_1 + x}\right)^{n_1} + \bar{p}_{0_2}\left[\frac{l_3\left(\frac{d_3}{d_2}\right)^2 + l_2}{l_3\left(\frac{d_3}{d_2}\right)^2 + l_2 - x}\right] - \phi \tag{6}$$

Equation (6) may be rewritten more conveniently in the form

$$\frac{l_2}{g}\frac{d^2\eta}{dt^2} = \bar{p}_{0_1}\left(\frac{\lambda}{\lambda + \eta}\right)^{n_1} + \bar{p}_{0_2}\left(\frac{R + 1}{R + 1 - \eta}\right)^{n_2} - \phi \tag{7}$$

where $\eta = x/l_2$, $\lambda = l_1/l_2$, and R is the ratio of driver-chamber volume to piston-tube volume $(l_3 d_3^2)/(l_2 d_2^2)$

Assuming ϕ is a constant (refer to subsection C), Eq. (6) is integrated once, subject to the initial conditions that $d\eta/dt = 0$ when $\eta = 0$ and gives the expression

$$\left(\frac{l_2}{2g}\right)\left(\frac{d\eta}{dt}\right)^2 = \frac{\bar{p}_{0_1}\lambda^{n_1}}{1 - n_1}\left[(\lambda + \eta)^{(1-n_1)} - \lambda^{(1-n_1)}\right] + \bar{p}_{0_2}\frac{(R + 1)^{n_2}}{1 - n_2}\left[(R + 1)^{(1-n_2)} - (R + 1 - \eta^{(1-n_2)}\right] - \phi\eta \tag{8}$$

To avoid the complicated mechanical problem of absorbing the impact of the piston at the end of its stroke and to insure that all possible kinetic energy is extracted from the piston by the driver gas, the piston-driving pressure must be adjusted to make the piston velocity equal to zero when it reaches the face of the driver-chamber.

Setting $\eta = 1$ and $d\eta/dt = 0$ in Eq. (8), the piston-driving pressure for no impact is found to be

$$(\bar{p}_{0_1})_{\text{zero impact}} = \frac{n_1 - 1}{\lambda^{n_1}\left[(\lambda + 1)^{(1-n_1)} - \lambda^{(1-n_1)}\right]} \times \left\{\frac{\bar{p}_{0_2}(R + 1)^{n_2}}{1 - n_2}\left[(R)^{(1-n_2)} - (R + 1)^{(1-n_2)}\right] - \phi\right\} \tag{9}$$

Substitution of Eq. (9) into Eq. (8) gives the final expression for piston velocity

$$\frac{l_2}{2g}\left(\frac{d\eta}{dt}\right)^2 = \bar{p}_{0_2}\frac{(R+1)^{n_2}}{n_2 - 1}$$

$$\times\left\{\frac{\left[(\lambda+\eta)^{(1-n_1)} - \lambda^{(1-n_1)}\right]\left[R^{(1-n_2)} - (R+1)^{(1-n_2)}\right]}{(\lambda+1)^{(1-n_1)} - \lambda^{(1-n_1)}}\right.$$

$$\left. + (R+1)^{(1-n_2)} - (R+1-\eta)^{(1-n_2)}\right\}$$

$$- \phi\left\{\eta - \frac{(\lambda+\eta)^{(1-n_1)} - \lambda^{(1-n_1)}}{(\lambda+1)^{(1-n_1)} - \lambda^{(1-n_1)}}\right\} \qquad (10)$$

B. *Solution of the Isentropic Equation: Stroke Time*

For the moment, assume that the friction parameter ϕ is small enough, compared with the other terms in Eq. (10), to be neglected. In this instance Eq. (10) may be rewritten as

$$\left(\frac{d\eta}{dt}\right)^2 = \frac{2g\,p_{0_2}\,(R+1)^{\gamma_2}}{\rho a\, l_2\,(1-\gamma_2)}\left\{\frac{(\lambda+\eta)^{(1-\gamma_1)} - \lambda^{(1-\gamma_1)}}{(\lambda+1)^{(1-\gamma_1)} - \lambda^{(1-\gamma_1)}}\right.$$

$$\times\left[R^{(1-\gamma_2)}\,(R+1)^{(1-\gamma_2)}\right] + (R+1)^{(1-\gamma_2)}$$

$$\left. - (R+1-\eta)^{(1-\gamma_2)}\right\} \qquad (11)$$

where the compression and expansion processes are assumed to be isentropic.

Since the total stroke time is related to the instantaneous velocity by the expression

$$T_s = \int_0^1 \left(\frac{d\eta}{dt}\right)^{-1} d\eta \qquad (12)$$

the stroke time can be studied by examining Eq. (11). It is immediately evident that

$$T_s \sim \left(\frac{\rho\, a\, l_2}{g\, p_{0_2}}\right)^{1/2} \qquad (13)$$

For minimum stroke time the piston density and length must be kept small, and the initial pressure in the driver-chamber should be as high as possible. The dependence of T_s on $l_2^{1/2}$ indicates that, for a given volume requirement, the stroke time decreases as d_2/d_3 increases.

All of the preceding discussion was merely qualitative. To obtain quantitative results, it is necessary to integrate Eq. (10) and (12). It will be useful to interpret the results of this integration in terms of the nondimensional parameter implied in Eq. (13)

$$\bar{T} = T_s\left(\frac{g\, p_{0_2}}{\rho\, l_2\, a}\right)^{1/2} \qquad (14)$$

The integration was performed on an IBM 7090 computer using the Adams-Moultan integration procedure with initial values calculated by the Runge-Kutta method. The results of this integration are shown in Fig. 2 and are based on isentropic compression and expansion processes for a monatomic gas.

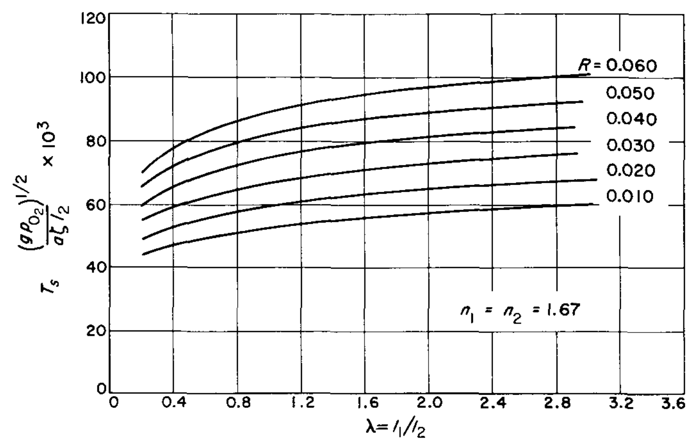

Fig. 2. Piston stroke time as a function of the ratio of reservoir to piston-tube length with compression ratio as a parameter

It was found that the stroke time decreases as the ratio of driver to piston-tube volume decreases. Furthermore, decreasing λ also decreases stroke time. This advantage is offset by the fact that the final position of the piston becomes highly sensitive to small fluctuations in initial driving pressure as λ becomes small. To insure sufficient control of the piston, then, it is necessary to keep λ, at least, of the order of 0.10.

To give an indication of the magnitude of the stroke time under various conditions, Fig. 3 illustrates the operation of a simple aluminum piston 6 in. long operating in a 12-in.-D piston tube, with the initial volume of piston-driving gas equal to 20% of the total piston-tube volume. The final driver pressure in all cases was 1,000 atm and the driver length was chosen so that the head of the reflected expansion wave would overtake the contact surface behind a Mach-40 shock front at a point 80 diameters from the diaphragm of a constant area, He/air, shock tube.

Figure 4 shows the effect of γ on the stroke time. It is evident that the total stroke time is nearly independent of the properties of the piston-driving gas while the properties of the driver gas are quite important. It takes nearly twice as long to compress a diatomic gas as it does to compress a monatomic gas, partly because of the nature of the piston motion and partly because of the increased stroke length required to reach the required temperature with a driver gas, which is less sensitive to volume change.

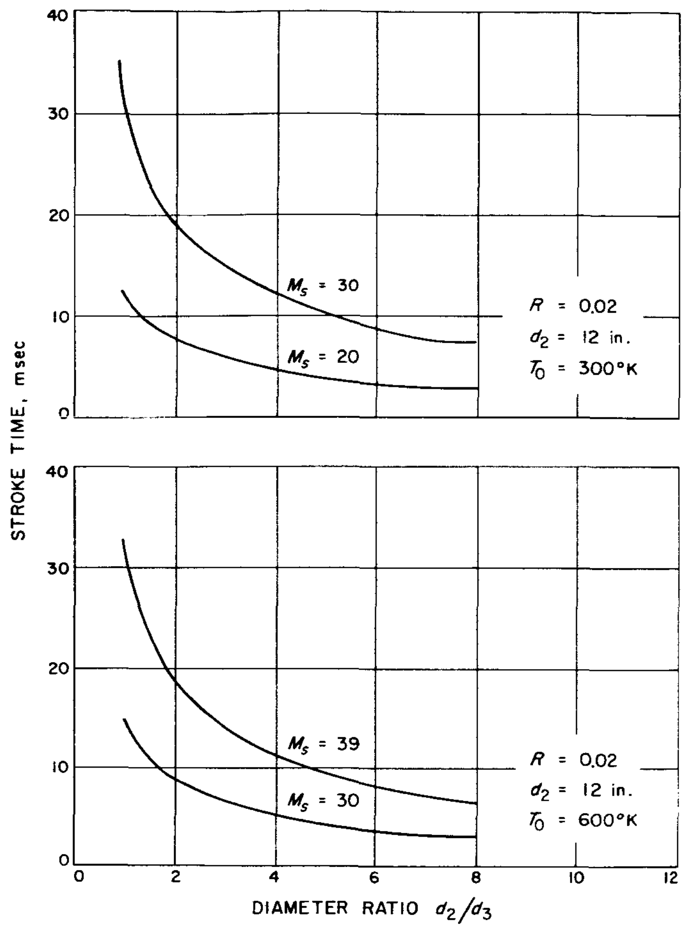

Fig. 3. Stroke time of a 6-in.-long aluminum piston compressing helium to 1000 atm, with initial volume of piston-driving gas 20% of total piston tube volume

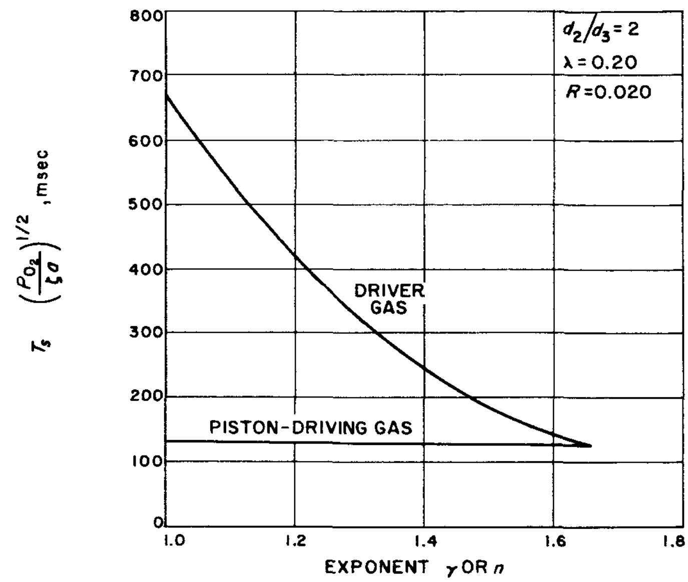

Fig. 4. Effect of polytropic exponent on stroke time (with n or γ assumed constant during the stroke)

C. *Nonisentropic Motion: Stroke Time*

In reality, of course, the expansion of the piston-driving gas and the compression of the gas in the driver-chamber are not isentropic processes. The driver gas loses heat to the walls and the piston-driving gas is heated by the walls and must overcome the sliding friction of the piston. To determine the effect of the nonisentropic nature on the stroke time, an estimate must be made of the polytropic exponents n_1 and n_2, which characterize the thermodynamic processes. This is done by first estimating the energy losses and then relating these losses to decreases in n. To do this, consider two equal masses of gas, each at the same initial state, which undergo a change of volume from V_0 to V. If only one of the gases goes from V_0 to V isentropically, the ratio of the final internal energies for calorically perfect gases is

$$\frac{E}{E'} = \frac{C_v\,T\,\rho_g\,V}{C_v'\,T'\,\rho_g'\,V'} \tag{15}$$

where the superscript denotes a nonisentropic change. As long as there is no significant ionization of the gas

$$C_v' = C_v$$

therefore, Eq. (15) reduces to

$$\frac{E}{E'} = \frac{T}{T'} \tag{16}$$

Now, for a specific process which starts at V_0 and ends at V the polytropic law gives

$$\frac{T}{T_0} = \left(\frac{V_0}{V}\right)^{n-1} \tag{17}$$

Substitution of Eq. (17) into (16) gives

$$\frac{E}{E'} = \frac{(V_0/V)^{\gamma-1}}{(V_0/V)^{n-1}} \tag{18}$$

Solving Eq. (18) for n results in

$$n = \gamma - \frac{\ln \dfrac{E}{E'}}{\ln \dfrac{V_0}{V}} \tag{19}$$

Now, an estimate of an energy loss can be converted into a corresponding change in the polytropic exponent which characterizes the process. Figure 4 shows the effect of n on the total stroke time. It must be remembered that these calculations are for a time-averaged value of the exponent. It can be shown, however, that almost all the energy loss occurs in the last portion of the stroke, and a change in the instantaneous value of n at that point would have a small effect on T_s.

Finally, the effect of friction on piston motion must be determined. Two assumptions regarding friction were made previously: (1) the frictional force is constant, and (2) its effect is small compared with that of the pressure forces. The first assumption is verified by experimental evidence (Ref. 4). The degree of approximation introduced by the second depends upon the piston diameter and pressure levels.

The frictional forces enter into the equation of motion in the form of the friction parameter ϕ. Since this parameter is inversely proportional to d_2^2, a pronounced frictional effect would not be expected for large piston diameters. As a matter of fact, when $d_2 = 12$ in., for a total frictional force of 1600 lbs (requiring a great amount of deliberate effort to get so high a frictional force) the total time changes by less than one-half of one percent. As the diameter decreases the effect becomes more pronounced, but for a piston diameter as small as 4 in. the total change in stroke time is still only 6% with a 1600-lb frictional force.

Figure 5 shows the effect of friction on stroke time. As predicted, the effect is always small, having the largest influence at lower diameters. An interesting result is that friction shortens stroke time. This is because the boundary condition requires the piston to move a specified distance. The increase in driving pressure required to overcome frictional retardation results in both higher accelerations and higher decelerations and, consequently, a shorter total stroke time.

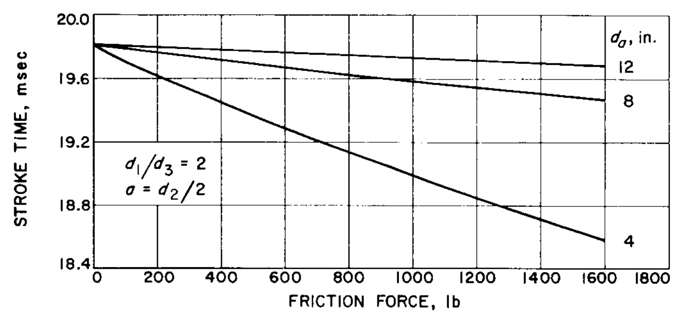

Fig. 5. Effect of friction on stroke time calculated for the $M_s = 39$ case of Fig. 3

III. HEAT LOSSES

There are quite obviously several loss mechanisms involved in the operation of the free-piston driver. There is only one, however, which is capable of removing energy from the driver gas in the piston tube: heat conduction to the walls of the tube. (Helium, the shock-tube driver gas considered in all of the following calculations, is incapable of radiating significant amounts of energy under the conditions of temperature and pressure to be encountered during piston-driver operation.) The heat conduction problem is a complicated one. Since the temperature of the gas in the center of the tube is different from the temperature of the gas at the wall by as much as an order of magnitude, the thermal conductivity must obviously be considered as variable. This fact alone precludes a closed-form solution of the problem. Add to this the fact that the gas temperature and pressure are changing with time, as is the area which is exposed to the high temperature gas, and the only alternative solution is a numerical integration to determine the heat loss.

The motion of the piston from the rear of the piston tube to the face of the driver-chamber is divided into a finite number of increments. During each increment, the piston is assumed to move impulsively from some position x to a new position $x + \delta x$ where it remains for an amount of time δt, where

$$\delta t = \int_{x}^{x+\delta x} \frac{1}{dx/dt}\, dx \tag{20}$$

The equation of motion of the piston was derived in section II and leads to an expression for dx/dt as a function of the driver geometry, the gas properties, and x.

The incremental heat loss during the interval δt is given by the expression

$$\delta Q = k_w\, A \left(\overline{\frac{\partial T}{\partial y}}\right)_w \delta t, \tag{21}$$

where $(\overline{\partial T/\partial y})_w$ is the time averaged value of the temperature gradient at the wall. Since the temperature gradient varies as $(t)^{-\frac{1}{2}}$, $(\overline{\partial T/\partial y})_w$ is the value of the temperature gradient evaluated at a time t^*, where t^* is given by

$$t^* = \left[\frac{\delta t/2}{(t+\delta t)^{1/2} - (t)^{1/2}}\right]^2 \tag{22}$$

The temperature gradient is obtained by a numerical integration of the heat conduction equation using the method of Crank and Nicolson (Ref. 5). (Wall temperature is evaluated assuming a semi-infinite wall (Ref. 6); the thermal conductivity is assumed to vary as $T^{0.636}$ and is corrected for pressure by the method of Enskog (Ref. 7). Temperature profiles are carried from one step to the next and are magnified by the factor $(V_{previous}/V)^{\gamma-1}$ when the piston changes the gas volume.

Equations (20)–(22) along with Eq. (10) for piston velocity, form a complete statement of the problem and have been integrated using the Adams-Moultan method. Referring to Fig. 6, with l_1/l_2, l_3/l_2, and R fixed, and the condition that the piston come to rest at the face of the driver-chamber, the driver specific heat loss becomes a function of stroke time, size of the device d_2, and pressure level, typified by p_{0_2}. The effect of varying these parameters on the efficiency is shown in Fig. 6 for a representative set of conditions where $R = 0.02$, $l_1/l_2 = 0.11$, and $l_3/l_2 = 0.075$. This efficiency is represented as the specific heat loss and, also, as the effective polytropic exponent for the process.

The effect of piston diameter on heat losses is shown in Fig. 6(a). Although the influence of piston diameter on heat losses is small, it is of particular interest to know that increasing the diameter increases the efficiency of the compression process. Large-diameter drivers, then, are practical from a thermodynamic point of view.

The far more pronounced effect of stroke time is shown in Fig. 6(b). It can be seen that a reduction in stroke time (defined as the time for the piston to move from the rear of the piston tube to the face of the driver-chamber) results in a substantial increase in compression efficiency. The parameters which affect stroke time and a method for predicting the stroke time of a given driver were discussed in detail in the preceding section. It is of interest to recall that the stroke time increases with piston weight and decreases with initial pressure of the driver gas. The construction of light pistons, then, is obviously essential to efficient operation of the driver.

The effect of initial pressure of the shock-tube driver gas is shown in Fig. 6(c) for a fixed stroke time. The increase in thermal conductivity with pressure results in a corresponding increase in total heat lost during compression. The far more important specific heat loss,

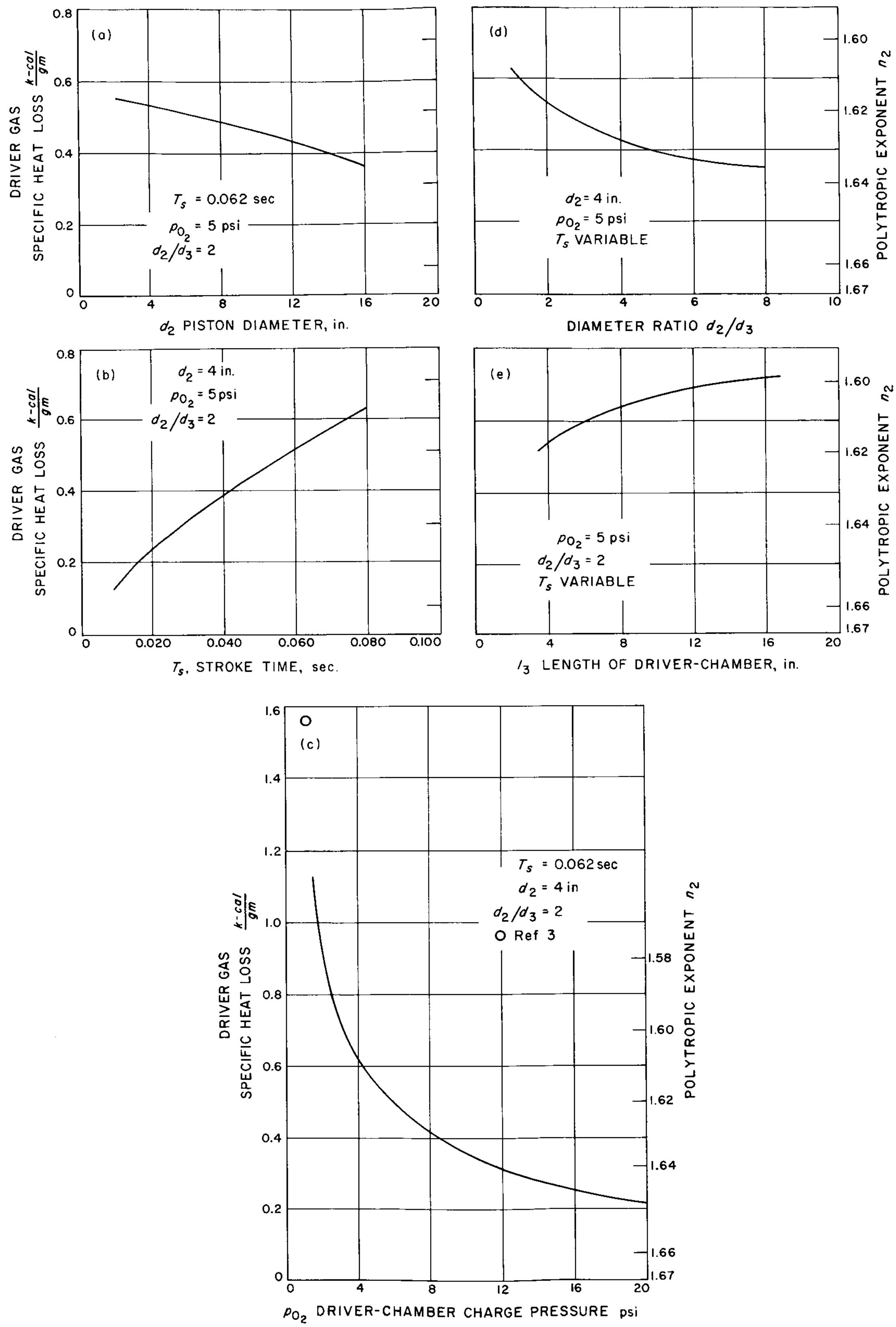

Fig. 6. Calculation of heat losses in a free-piston driver with a compression ratio of 50:1, nondimensional reservoir length λ, of 0.11, and nondimensional driver-chamber length L_3/L_2, of 0.075

depending upon mass as well as heat loss, decreases, however. This decrease in specific heat loss indicates more efficient compression. Since there is a large increase in compression efficiency with increasing pressure, and higher initial pressure decreases stroke time as well, it is most desirable to operate at the highest possible pressure levels. An experimental point from Ref. 3 for similar, but not identical, geometry is approximately 25% higher than the theoretical prediction.

Figures 6(d) and 6(c) show the effect of diameter ratio and length of the driver-chamber. They have been prepared as items of general interest, and it must be remembered that stroke time is variable in these two cases.

IV. OPERATION

Operation of the driver is relatively simple. Once the geometry has been fixed, Eq. (9) can be used to determine the ratio of piston-driving pressure to initial piston-tube charge pressure which will allow the piston to travel the entire length of the piston tube and come to a stop at the face of the driver-chamber. The final driver temperature, depending only upon the geometry, will be fixed and the final pressure is determined by the initial driver-chamber charge pressure and Eq. (4). The resulting shock strength may then be varied either by changing the initial driver-chamber charge pressure or by the addition of a heavy diluent gas, such as nitrogen, or by a combination of these two methods.

Knowing the effective polytropic exponent of the driver gas mixture and the geometry of the driver, the final driver temperature and pressure can be easily computed. This information, along with given driven tube conditions, is sufficient to calculate shock Mach number and required driver-chamber length by the usual methods. The criteria used to determine driver length is that the head of the reflected expansion wave should overtake the contact surface at the test section. Since real gas effects at shock Mach numbers in the range considered here lead to very large density ratios across the shock wave, it was considered sufficiently accurate to assume that the shock wave and the contact surface move with the same velocity.

Calculations of a constant-area shock tube with nitrogen in the driven tube at a pressure of 0.76 mm Hg are shown in Fig. 7 and 8. The piston compression ratio was 50:1 and the compression process was assumed to be isentropic. Figure 7 shows that the shock Mach number can be varied by a combination of changing the initial pressure in the driver-chamber and adding the nitrogen diluent. Figure 8 shows the advantage of varying the Mach number by adding the diluent gas. For any given shock Mach number, the allowable length of the driven tube, for a given driver length, increases substantially when the diluent gas is added. In addition, Fig. 7 shows that for a given shock Mach number the initial pressure in the driver-chamber is raised by the addition of a diluent. Since this higher pressure operation decreases heat losses during compression, operation of the driver, using a diluent gas, is highly desirable.

The amount of diluent in the driver gas may be varied so as to make the required driver-chamber length a minimum for any given Mach number. Figure 9 shows such a variation for a ratio of driven-tube length to driver length of 44:1. Figures 7 and 9 provide a complete operational guide for an ideal compressor with a compression ratio of 50:1.

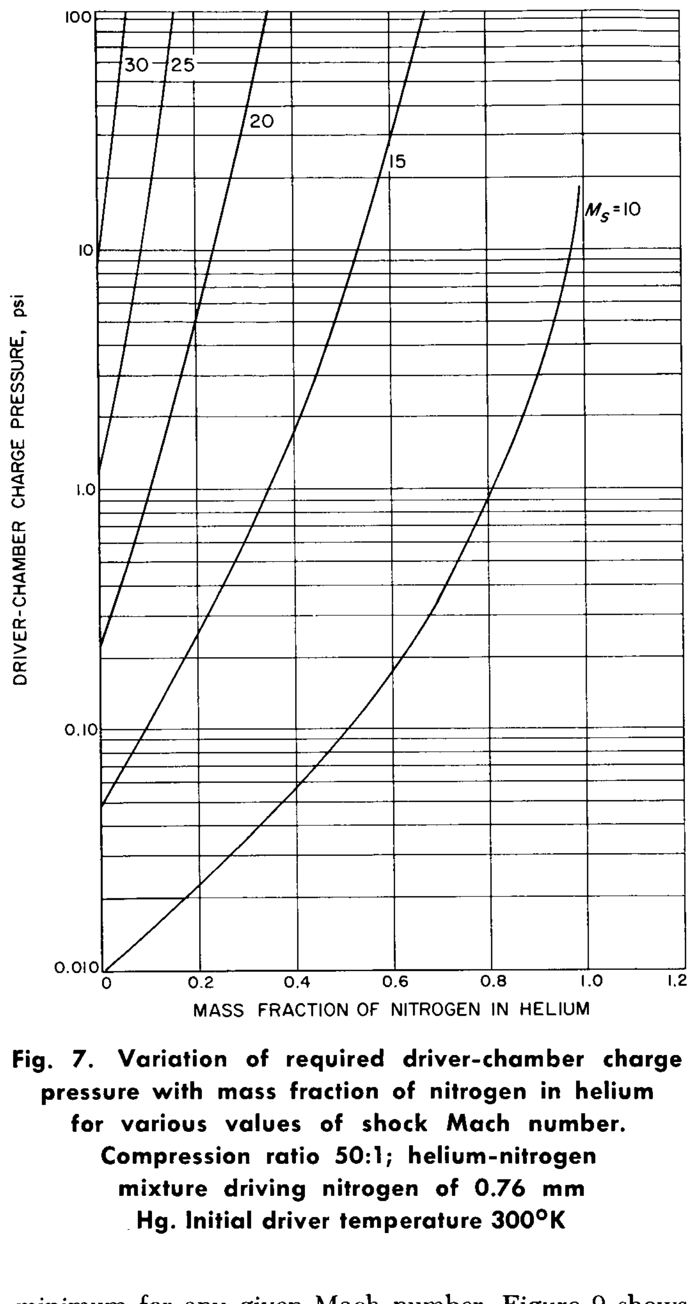

Fig. 7. Variation of required driver-chamber charge pressure with mass fraction of nitrogen in helium for various values of shock Mach number. Compression ratio 50:1; helium-nitrogen mixture driving nitrogen of 0.76 mm Hg. Initial driver temperature 300°K

There is one point of caution to be considered concerning the driver-chamber length calculations. The equations used were derived under the assumption of the

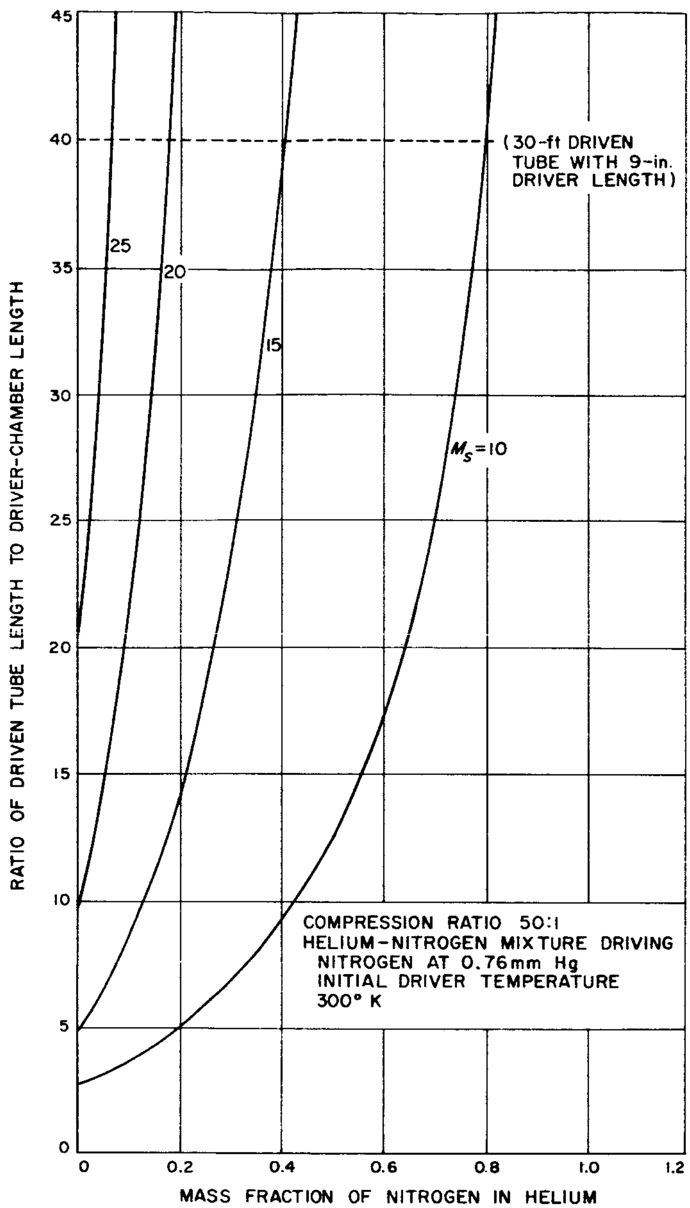

Fig. 8. Ratio of driven-tube length to required length of driver-chamber as a function of mass fraction of nitrogen diluent in helium driver gas

existence of a centered expansion wave. At the high driver-gas temperatures encountered in the production of strong shock waves, the time for the expansion wave to traverse the driver-chamber is the same order of magnitude as the diaphragm opening time. Under these conditions the assumption of a centered expansion wave is no longer valid. The current state of the art, however, does not allow an exact determination of the wave pattern produced by a finite diaphragm opening time. There is experimental evidence to indicate that the wave

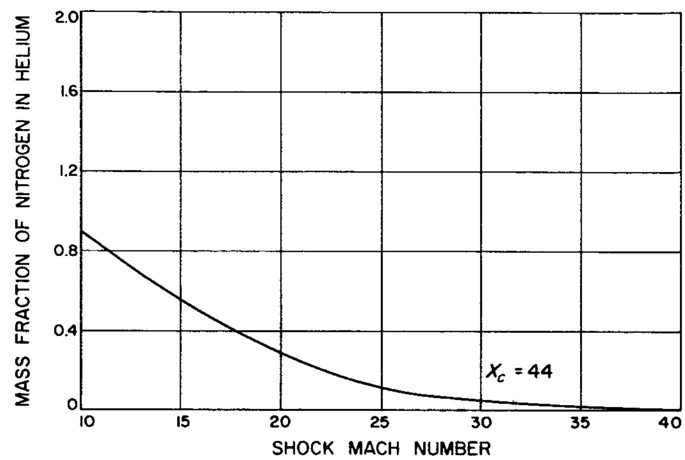

Fig. 9. Variation of nitrogen mass fraction in helium driver gas with shock Mach number for a ratio of driven tube length to driver length of 44

pattern at a considerable distance from the diaphragm is close to that predicted by the ideal theory; and so, in the absence of a more accurate technique, the centered expansion wave assumption has been made.

There remains now the problem of determining the effect of piston bounce. Although the piston velocity is zero at the end of the stroke, it is subjected to a large negative acceleration and immediately starts to move rearward. This reverse motion of the piston starts the instant that the diaphragm is ruptured. The retracting piston causes expansion waves to travel forward and reduce the strength of the shock wave. The resulting wave pattern is shown in Fig. 10. Note that only those waves leaving the piston face before the diaphragm expansion wave arrives will reach the contact surface before the test section, since the criterion for determining driver length is to allow the head of the reflected expan-

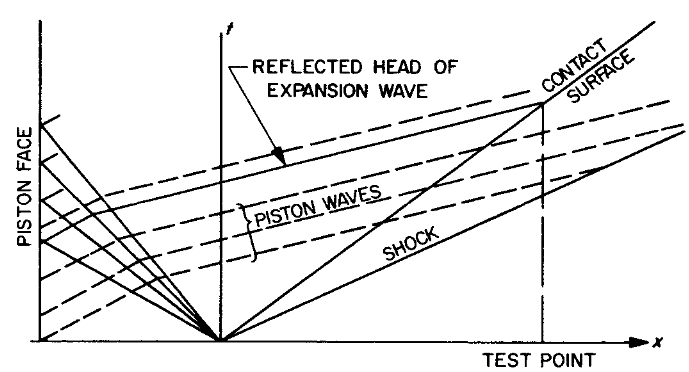

Fig. 10. x — t diagram showing the effect of piston bounce

sion wave to intersect the contact surface at that point. For the case of a shock Mach number of 39 in the facility of Fig. 3, only that motion which takes place in the first 45 μsec after the diaphragm ruptures will be felt in the test-gas slug. The motion analysis described in section II indicates that the volume increase during this time interval is less than 0.1%, and that the effect of piston bounce on the test slug is insignificant.

V. INCREASING THE PERFORMANCE OF A FIXED FACILITY

A. Preheating

The relation between driver-gas temperature change and volume change is

$$\frac{V_0}{V} = \left(\frac{T}{T_0}\right)^{\frac{1}{n-1}}$$

where the subscript denotes conditions before compression. For a given size driver-chamber, it is necessary to keep the ratio V_0/V small in order to keep the total size of the facility reasonable. This necessitates keeping T/T_0 as small as possible. For a given shock Mach number and driver pressure, the temperature T will be fixed. If the initial temperature T_0 is raised, therefore, the total piston tube volume requirement can be decreased.

For example, consider a final driver temperature of 8000°K and $n = 1.67$. Raising the initial driver-gas temperature from 300 to 600°K decreases the piston-tube length by a factor of 2.82 if the area is held constant. If a maximum driver pressure of 1000 atm is assumed (dictated by structural limitations), then, for a diameter ratio d_2/d_3 of 2 and a 9-in. chamber length, a total compression length of 28 ft is required to produce a Mach-40 shock in 10^{-3} atm of nitrogen. If the gas is preheated to 600°K the required length decreases to only 10 ft. From another point of view, for a given length compression stroke, say 10 ft, with preheat the shock Mach number will be 40; without preheat it will be 29.

The only practical method of preheating the gas is to heat the entire driver. For a representative 12-in.-ID driver 10 ft long 60 kw-hr are required to raise the temperature to 600°K. With only a 1-in. layer of standard commercial insulation the driver could be maintained at 600°K by only 250 w of continuous power. There are, however, certain practical problems involved with the heated driver. Handling and changing diaphragms will be difficult; the driven tube will have to be cooled in the vicinity of the diaphragm, and instrumentation of the driver will be difficult. For these reasons, alternate schemes of increasing performance are discussed in the remainder of this section.

B. Addition of a Buffer Section

An additional chamber and diaphragm could be inserted between the driver and driven tubes, thereby adding a buffered capability to the system. The driver would send a shock through the buffer chamber which would reflect off the buffer diaphragm and leave the buffer gas with no velocity, a high temperature and a high pressure. Then, the buffer diaphragm would be broken mechanically and the final shock sent down the driven tube.

The buffer section has two distinct disadvantages. The first is the degree of mechanical complication which it adds to the system; the second is that the degree of improved performance is the same as that gained by preheating, although the driver dimensions are increased substantially (Ref. 8).

C. Construction of the Driver

If the driver diameter is reduced the shock strength, for a given driver temperature and pressure, will also be reduced. This decrease, however, is more than offset by the far higher driver temperature created by the resulting larger compression ratio. Therefore, by inserting a hollow tubular sleeve into the driver section, the driver diameter will be decreased and the shock Mach number will increase.

As a numerical example, consider the case of the 10-ft-long, 1-ft-D piston tube discussed in section V. With a 6-in.-D driver (assuming a 6-in.-D driven tube) the shock Mach number without preheating was 29. If the driver diameter were now reduced to 3 in., then the resultant shock Mach number would be 38.

D. Combinations

The driver gas may be preheated and the chamber diameter constricted as well. The effects are additive, since they each tend to raise the final driver temperature. Their capabilities are not unlimited, however; if the temperature exceeds 10,000°K, an appreciable amount of energy will go into ionizing the gas, and this energy does not increase the shock strength. The practical limit to increasing Mach number then appears to be of the order of Mach-40 if the pressure ratio is limited to 10^6.

VI. CONCLUSIONS

An analysis of the free-piston driver technique shows it to be a highly promising method for the production of high-enthalpy shock-tube flows. The free-piston driver can be fully competitive with conventional arc-discharge drivers while offering the advantages of a uniformly heated driver gas, which is relatively free of heavy contaminants.

The heat losses associated with the compression process can be minimized by operating at the highest possible pressure levels with light pistons. The importance of this statement cannot be overemphasized. Isolated experimental evidence indicates that operation at low pressure levels with heavy pistons results in losses which are greater than those predicted theoretically, while the reverse gave results which are quite close to theoretical predictions. It is found that the driver increases in efficiency as the total size increases and that the only effect of friction is to increase the pressure requirement on the piston-driving gas.

For a fixed free-piston facility a complete range of Mach numbers is available through the use of a heavy diluent gas in the driver, preheating the driver gas, and/or increasing the compression ratio by constricting the diameter of the driver chamber. Thus, the free-piston driver promises to be a powerful and flexible tool for high-enthalpy research.

NOMENCLATURE

a	piston length
A	surface area exposed to hot gas
c_v	specific heat at constant volume
d	diameter
E	total energy content
f	frictional force on piston
F	total force on piston
g	acceleration due to gravity
k	thermal conductivity
ℓ	chamber length
m	piston mass
M_s	shock Mach number
n	polytropic exponent defined by Eq. (2)
p	pressure
Q	heat loss
R	ratio of driver-chamber volume to piston-tube volume
t	time
t^*	time defined by Eq. (22)
T	temperature
T_s	total time for piston to move from the rear of the piston tube to the face of the driver-chamber
V	volume
x	distance measured along axis of piston tube, starting at piston face
X_c	ratio of driven-tube length to driver length
y	coordinate normal to wall, measured from the wall
γ	ratio of specific heats c_p/c_v
λ	nondimensional length of piston-driving chamber ℓ_1/ℓ_2
η	nondimensional coordinate x/ℓ_2
ϕ	friction parameter, defined in section II
ρ	piston density
ρ_g	gas density
$(\ )_0$	initial conditions
$(\ )_1$	piston-driving chamber
$(\ )_3$	shock-tube driver-chamber
$(\ )_w$	conditions at the wall
$(\ )'$	nonisentropic change

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